Quantum information with atomic ensembles

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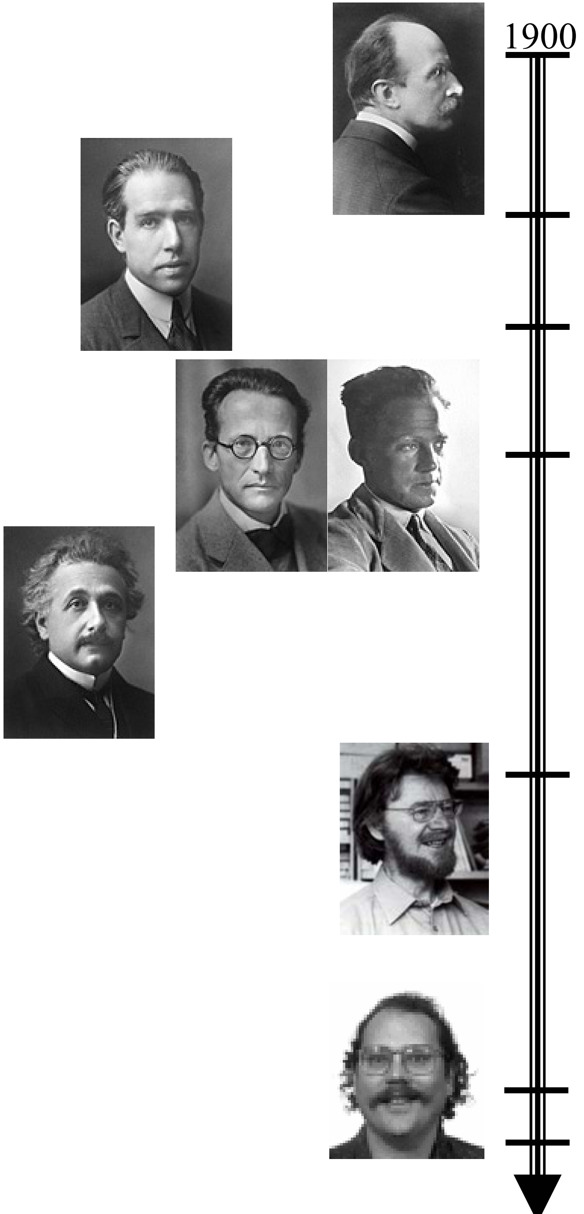
- R. Cote (Storrs)

SFB Coherent Control
€U TMR

Quantum Theory



- 1900 Planck:
- 1913 Bohr's model of the atom
- 1926 Schrödinger & Heisenberg
- 1936 Einstein – Podolski – Rosen
- 1963 Bell's inequalities
- 1993: Bennett: q. cryptography
- 1996 Shor's algorithm

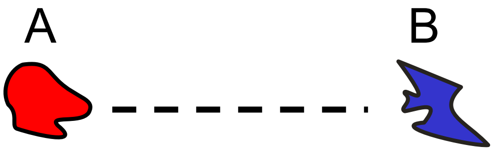
from paradox

to application

fotos: © R. Werner

Entangled States

- entanglement



states: $|0\rangle \otimes |0\rangle$

$|1\rangle \otimes |1\rangle$ ... product states

but also ...

$\frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$ ... entangled

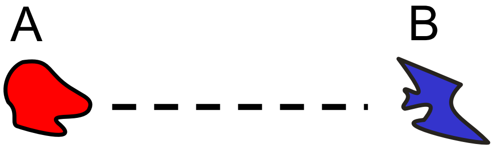
Schrödinger:
Verschränkung

- fundamental aspects of quantum mechanics
  - incompatibility of QM with LHVT
  - decoherence
  - measurement theory (?)
- applications
  - quantum communications & computing
  - precision measurement

Engineering Entangled States

We need ...

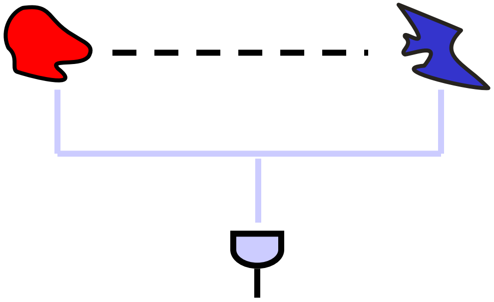
- “quantum engineering”



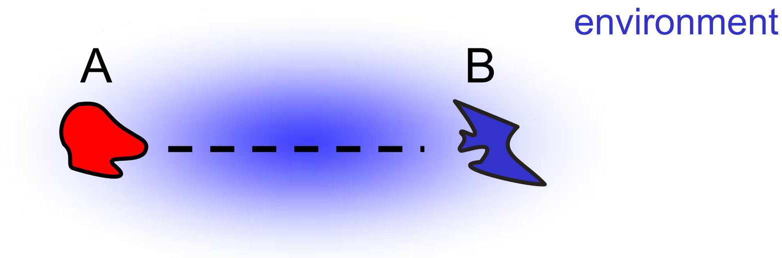
$$|a\rangle_A|b\rangle_B \rightarrow \sum c_{ab}|a\rangle_A|b\rangle_B$$

Hamiltonian evolution

- or: “quantum gambling”



- isolation



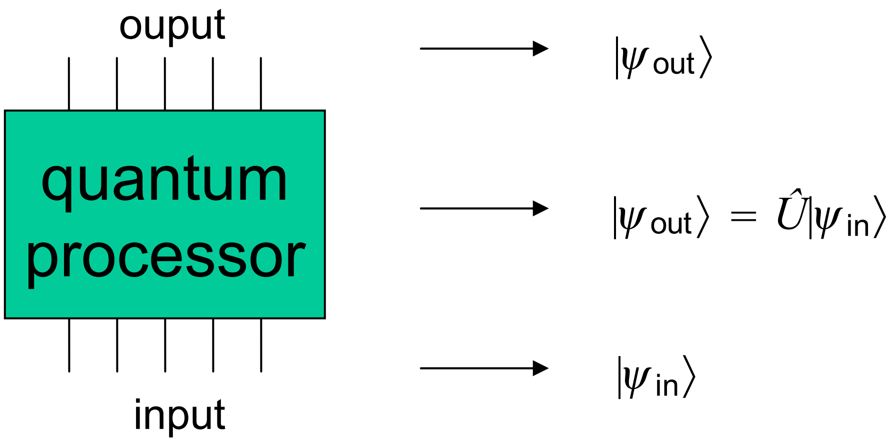
$$|\phi\rangle_A|\varphi\rangle_B|E\rangle \rightarrow |\Psi\rangle_{ABE}$$

$$\rho_{AB} = \mathrm{tr}_E|\Psi\rangle_{ABE}\langle\Psi| \neq |\Psi\rangle_{AB}\langle\Psi|$$

Quantum optical systems provide one of the best set-ups to create entangled states in a controlled way.

Quantum information processing

- quantum computing



ouput

quantum processor

input

$|\psi_{\text{out}}\rangle$

$|\psi_{\text{out}}\rangle = \hat{U}|\psi_{\text{in}}\rangle$

$|\psi_{\text{in}}\rangle$

quantum weirdness:

✓ superposition

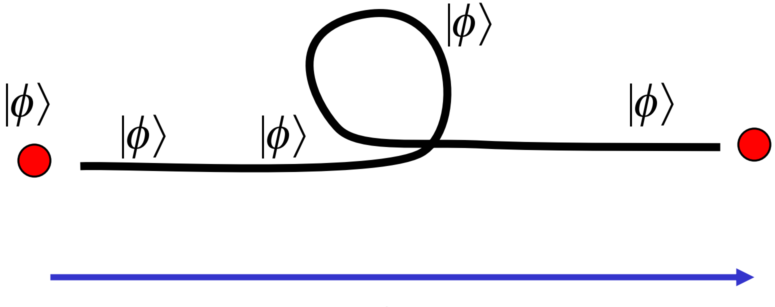
✓ entanglement

✓ interference

✓ nonclonability and uncertainty

✓ no decoherence!

- quantum communications



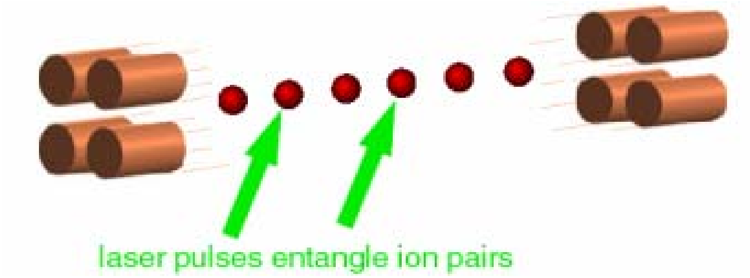
transmission of a quantum state

✓ teleportation

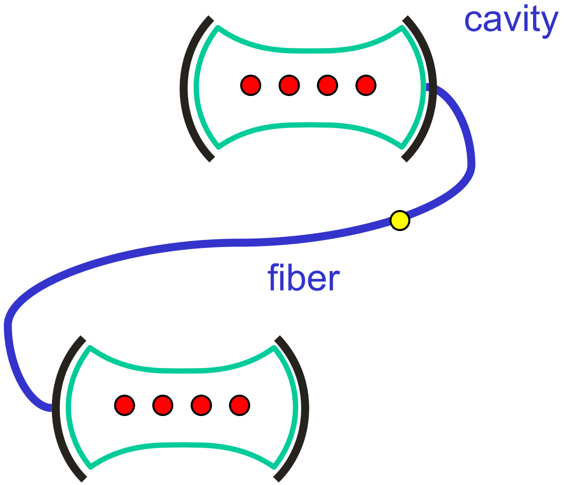
✓ crytography

Innsbruck proposals: examples ...

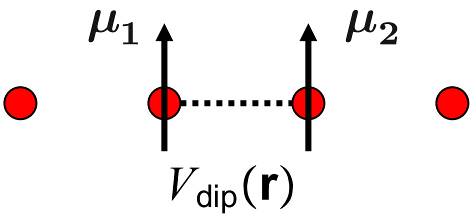
- ion traps '95



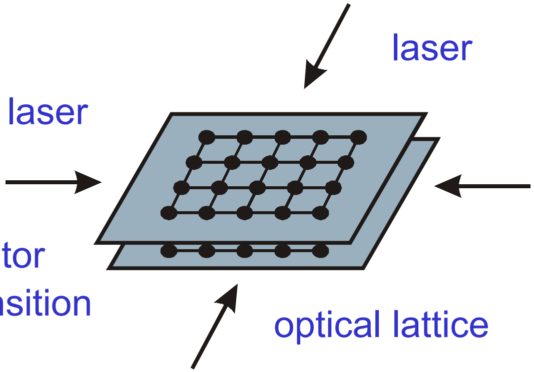
- optical cavity QED



- neutral atoms:



interacting Rydberg dipoles

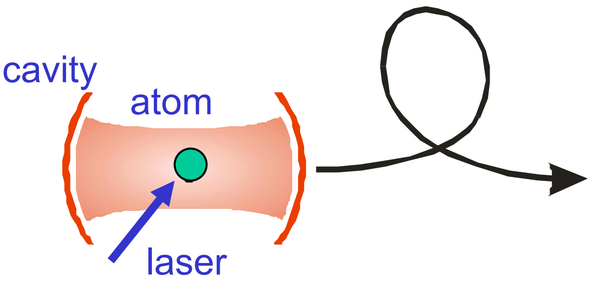


filling by Mott-insulator quantum phase transition from BEC

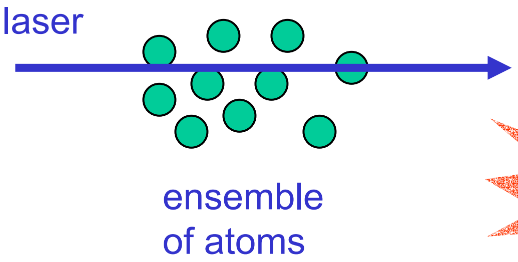
These systems realize manipulation on the *single* quantum level.

Is there a simpler way ? ... atomic ensembles

- features
  - so far: quantum computing and communications requires
    - ✓ single atoms and single photons
    - ✓ high-Q cavities



  - now: can we get away with ...
    - ✓ atomic ensembles?
    - ✓ free space or low Q-cavities?



simpler!

Our recent papers ...

- **Quantum repeaters with atomic ensembles and linear optics**
  L. M. Duan et al., Nature Nov 2001

- Quantum information with mesosocipc ensembles
  M. Lukin et al., PRL 2001

Rydberg dipole - blockade

- Teleportation with coherent light and atomic ensembles
  L. Duan et al. Dec PRL 2000
  exp.: E. Polzik et al., Nature Sep 2001

- ½-anyons in small Bose Einstein Condensates
  B. Paredes et al., Mar PRL 2001

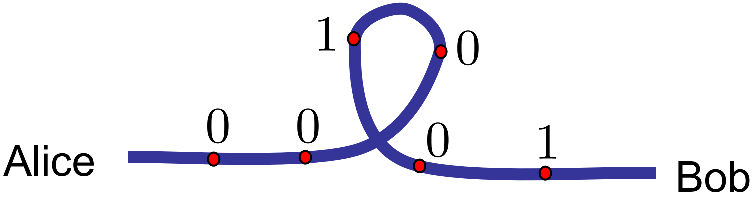
topological excitations

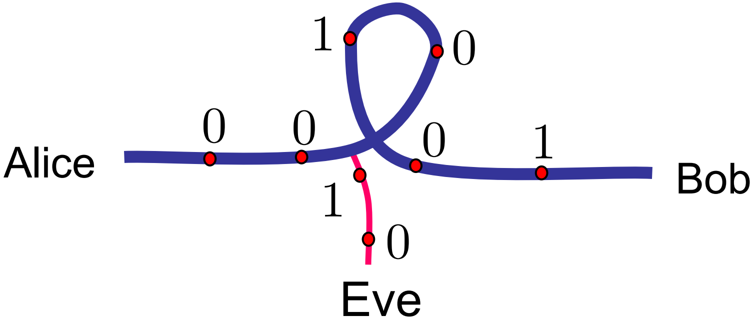
- Many particle entanglement with Bose Einstein Condensates
  A. Sorensen et al., Nature Jan 2001

precision measurement with
spin squeezing

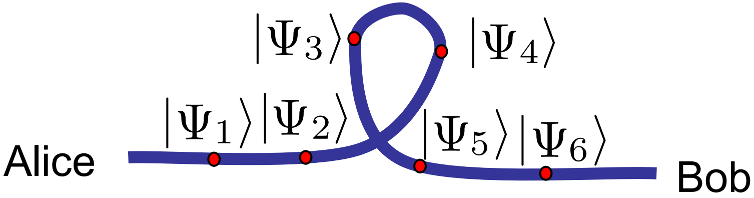
Quantum Communications

- classical communications

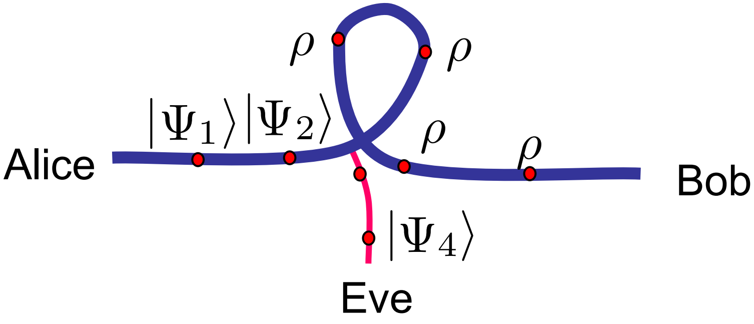




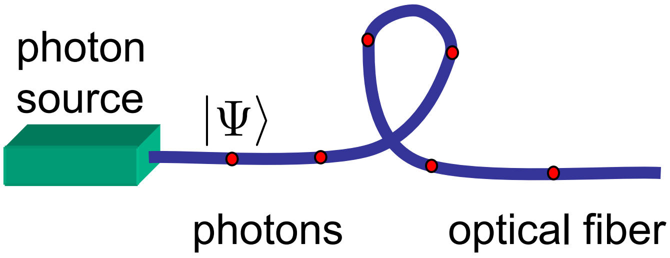
- quantum communications



✓ quantum networks
✓ cryptopgraphy



- implementation: photons



$|0\rangle = |\updownarrow\rangle$ vertical polarization

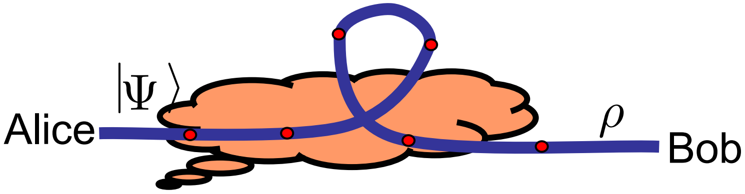
$|1\rangle = |\leftrightarrow\rangle$ horizontal polarization

- problem: decoherence

1. photons are absorbed:

- probability a photon arrives: $P = e^{-L/L_0}$
- quantum communication is limited to short distances (< 100 Km).

2. states are distorted:



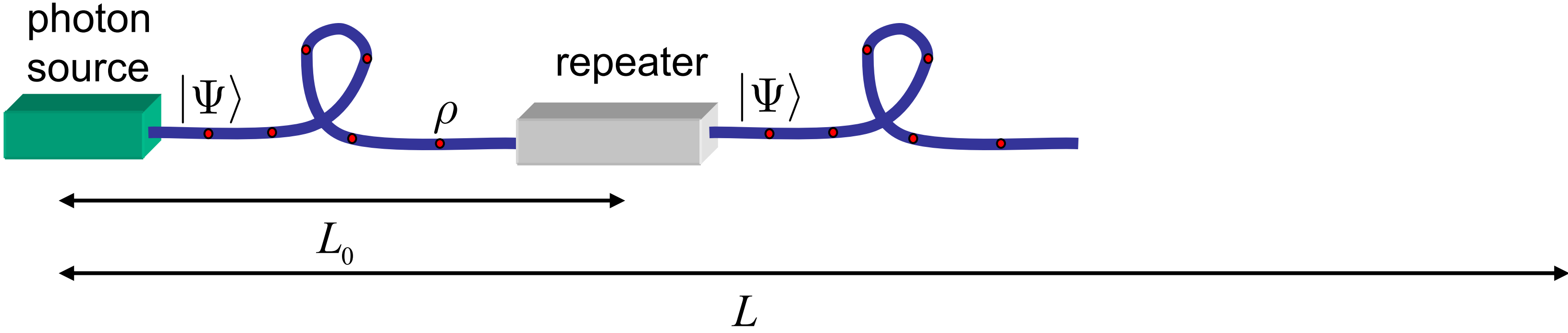
fidelity $F = \langle\Psi|\rho|\Psi\rangle < 1$

We cannot know whether this is due to decoherence or an eavesdropper.

... to regain fidelity we want:

Quantum Repeater

- goal



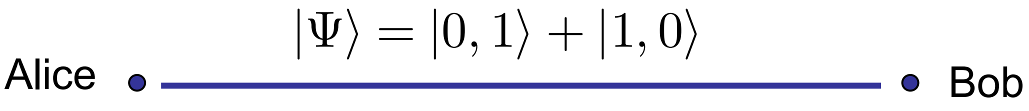
- properties:
  - overall fidelity $F = \langle\Psi|\rho|\Psi\rangle \simeq 1$
  - scaling of resources, e.g. communication time $\sim L^{\eta} < e^{L/L_0}$ with L length of communication channel

- Q.: concept of a repeater? implementation?

H. Briegel et al., PRL 98

Entanglement based quantum communication schemes

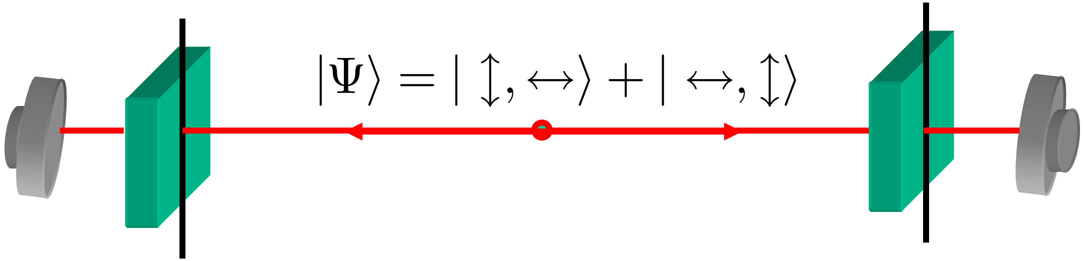
- entangled state



$$|\Psi\rangle = |0,1\rangle + |1,0\rangle$$

EPR correlations

- example: photon pair



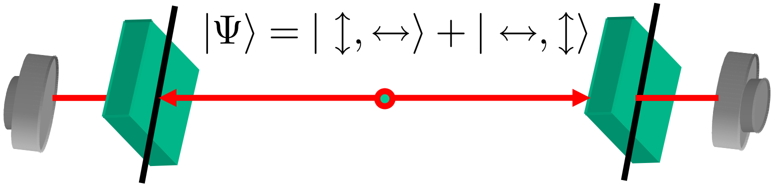
$$|\Psi\rangle = |\updownarrow,\leftrightarrow\rangle + |\leftrightarrow,\updownarrow\rangle$$

Applications

- secret communciation using entangled states: Ekert protocol

1. Check that particles are indeed entangled.

Correlations in all directions.



$$|\Psi\rangle = |\updownarrow,\leftrightarrow\rangle + |\leftrightarrow,\updownarrow\rangle$$

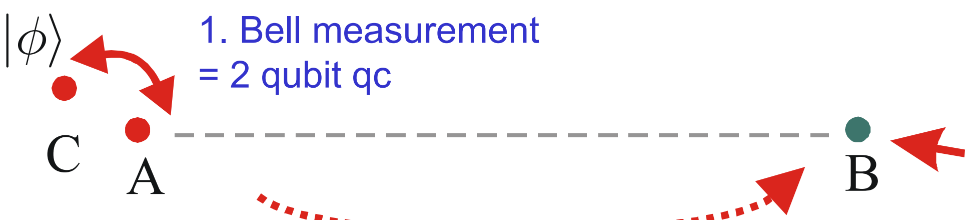
No eavesdropper present

2. Measure in A and B (z direction):

Alice	Bob
0	0
1	1
1	1
1	1
0	0

Send secret messages

- quantum teleportation



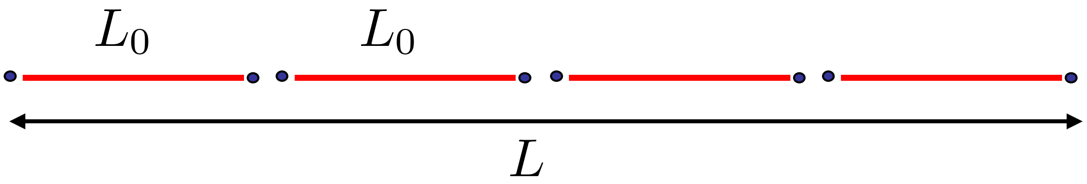
1. Bell measurement
= 2 qubit qc

2. classical communication

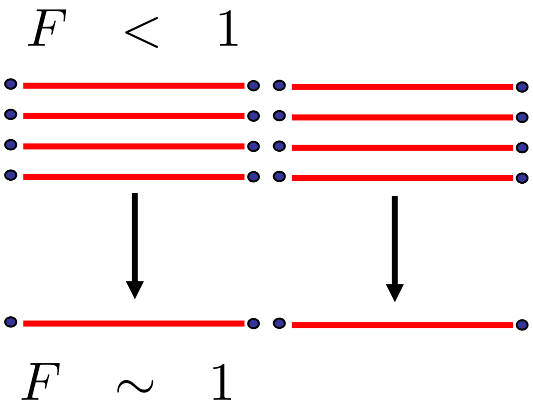
3. unitary operation

Quantum repeater: the concept

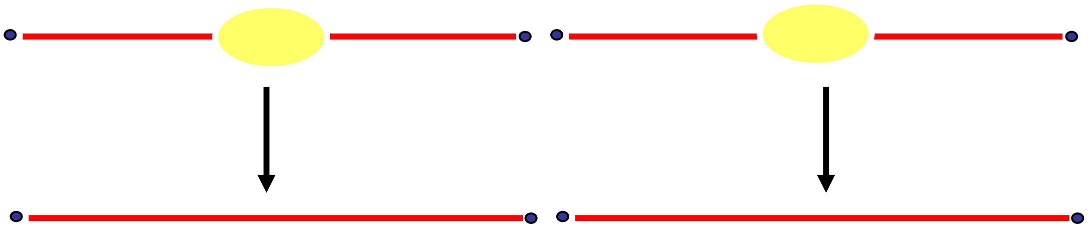
- goal: generate *long distance entangled pairs* with fidelity $F \sim 1$ in a small number of trials $\sim L^{\eta}$
- key ideas:
  - divide transmission channel into segments and generate pairs



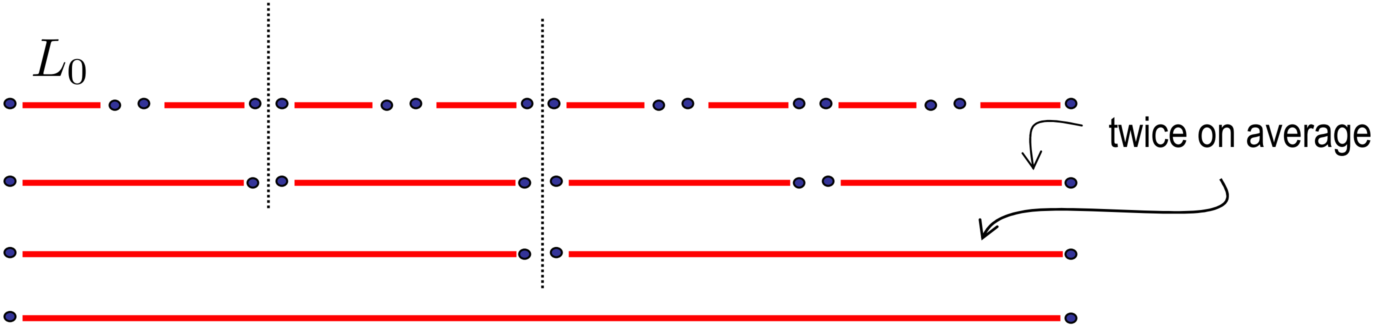
  - purification



- connect pairs to extend length by entanglement swapping



- putting all of this together



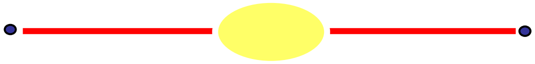
$$L = 2^n L_0$$

- efficiency:
  - number of elementary operations ~ L
  - with purification $\sim L^{\log_2 L}$

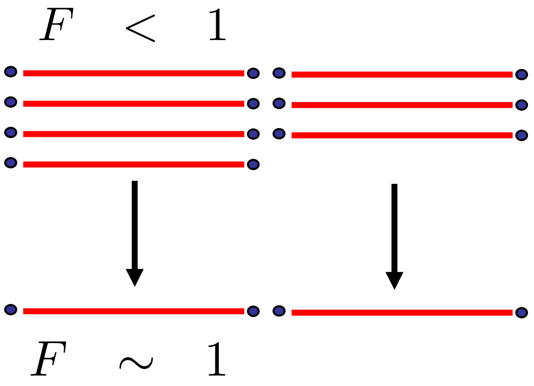
Quantum repeater: implementation

- requirements:
  - generate entanglement

  - store entangled states and perform collective local operations



- Remark: quantum memory is essential because purification protocols are probabilistic



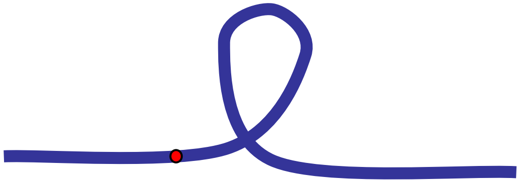
finished at different times!

- physical implementation

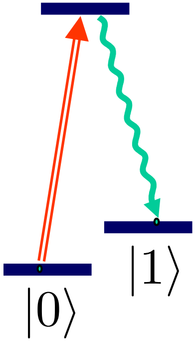
photons:
transmission

coupling ⟷

atoms:
storage

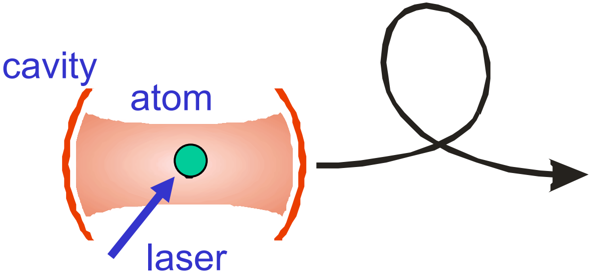


optical fiber

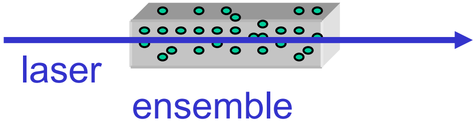


internal atomic levels

- normally one requires:
  - single atoms to store qubits
  - high Q cavities + strong coupling



- here: atomic ensembles, low Q-cavities or free space



Quantum repeaters with atomic ensembles

- Outline:
  - explain basic ideas with single atoms

    conceptually simple, but unrealistic
  - atomic ensembles

    simpler and works better

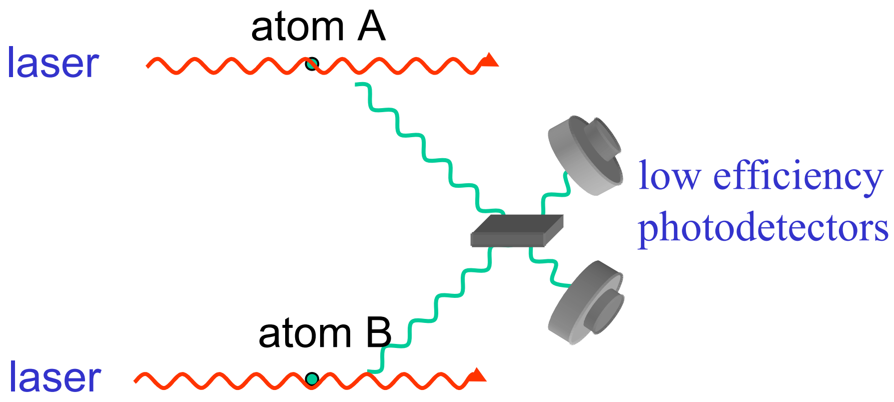
- issues:
  - entanglement generation
  - connection
  - decoherence and imperfections
  - applications

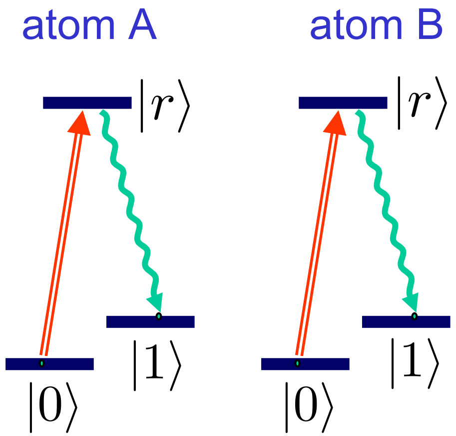
L. M. Duan et al., Nature Nov 2001

Cabrillo et al., PRA 99

Single atoms

- entanglement generation





- Weak (short) laser pulse, so that the excitation probability is small.

- If no detection, pump back and start again.

- If detection, an entangled state is created.

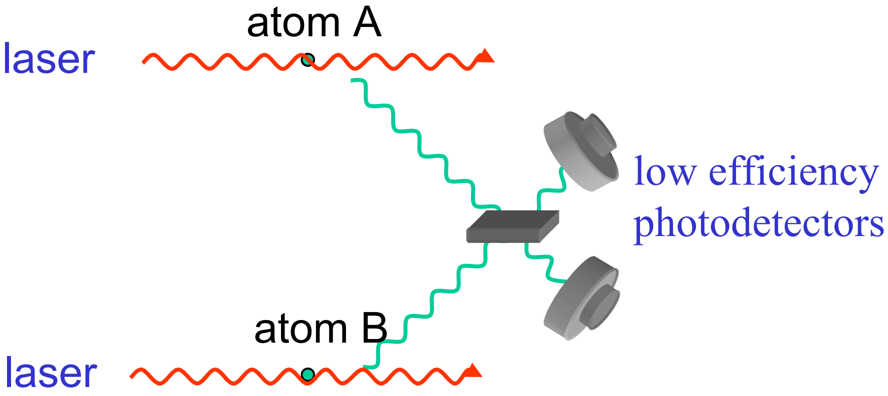
$$\sim |0,1\rangle + |1,0\rangle$$

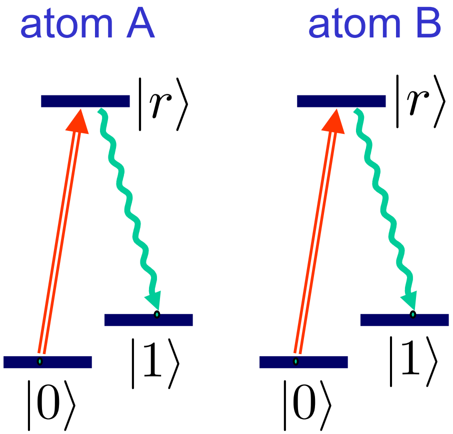
[Note: with photon loss an exponentially large number of repetitions in [ ]

Cabrillo et al., PRA 99

Single atoms

- entanglement generation

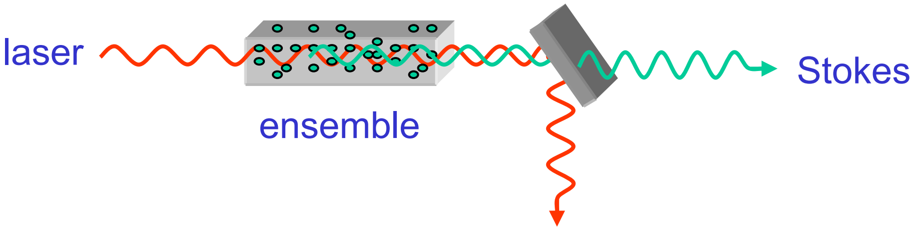


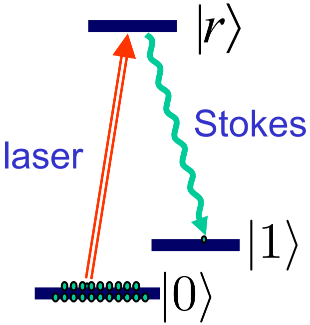


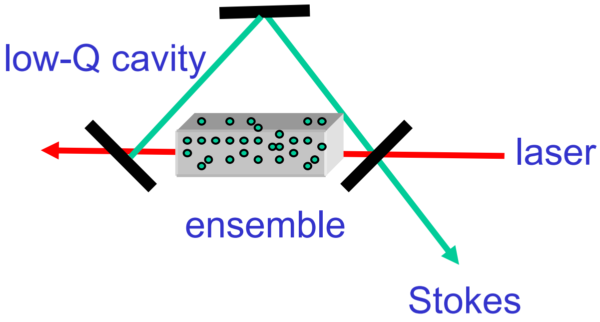
- Initial state: $|0,0\rangle|\mathrm{vac}\rangle$
- After laser pulse: $|0\rangle + \epsilon\,(|r,0\rangle + |r,0\rangle) + O(\epsilon^2)$
- Evolution: $|0,0\rangle|\mathrm{vac}\rangle + \epsilon \sum_k (b_k|0,1\rangle + a_k|1,0\rangle)\,|1_k\rangle + O(\epsilon^2)$
- Detection: $b_k|0,1\rangle \pm a_k|1,0\rangle \simeq |0,1\rangle \pm |1,0\rangle$

Atomic Ensembles

- system: cloud of cold atoms

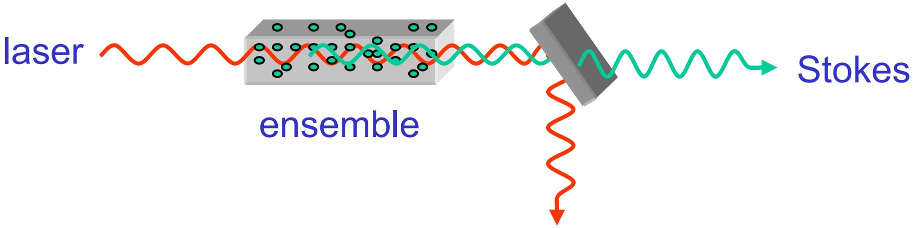


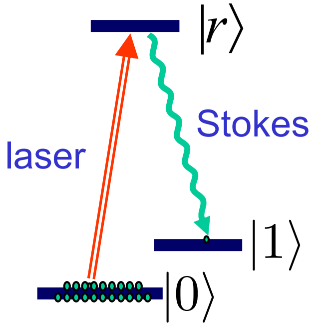




Atomic Ensembles

- system: cloud of cold atoms





- Raman process:

$$|0\rangle^{\otimes N} \equiv |\text{vac}\rangle \text{ (atomic ground state)}$$

$$\rightarrow \frac{1}{\sqrt{N_a}} \sum_{i=1}^{N_a} |0_1 \ldots 1_i \ldots 0_{N_a}\rangle \equiv a^\dagger |\text{vac}\rangle \quad \text{(single atomic excitation)}$$

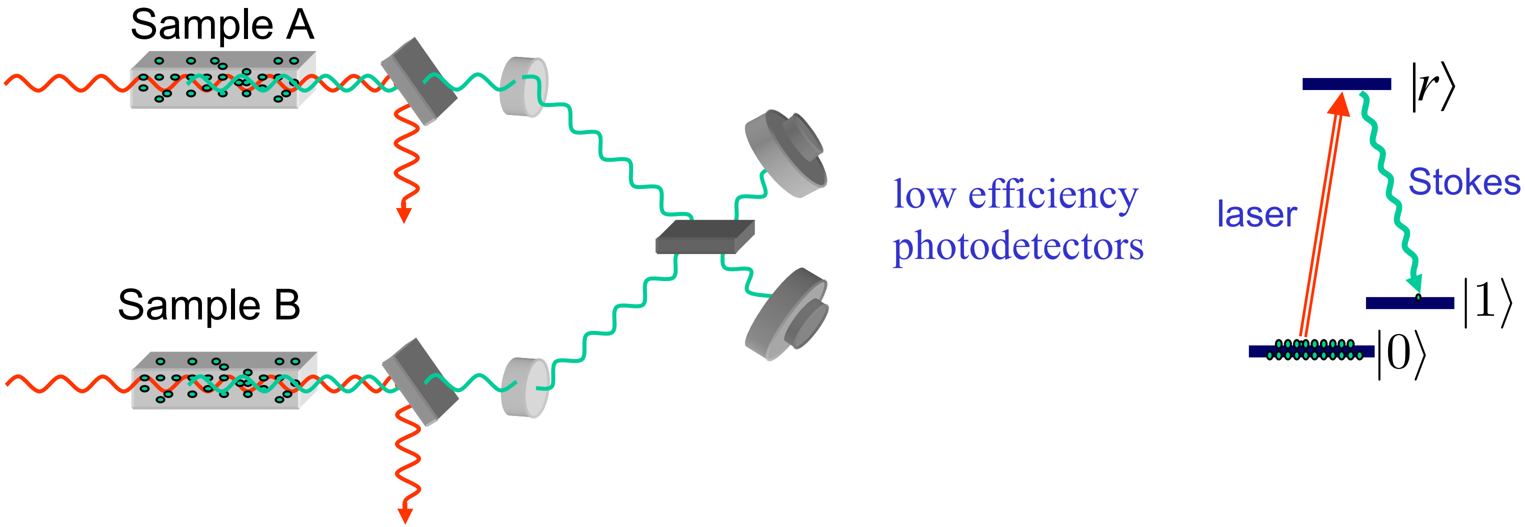
$$[a, a^\dagger] \approx 1$$

- state of atomic collective mode + Stokes photon

$$|\phi\rangle = |\text{vac}\rangle + \sqrt{p_c} a^\dagger c^\dagger_{\text{Stokes}} |\text{vac}\rangle + O(\sqrt{p_c}^2) \quad (p_c \ll 1)$$

... analogous to parametric

Generation of entanglement



low efficiency photodetectors

$$|\phi\rangle_A \otimes |\phi\rangle_B = \left(|\text{vac}\rangle_A + \sqrt{p_c}\, a^\dagger c_s^\dagger |\text{vac}\rangle_A\right) \otimes \left(|\text{vac}\rangle_B + \sqrt{p_c}\, b^\dagger c_s^\dagger |\text{vac}\rangle_B\right)$$

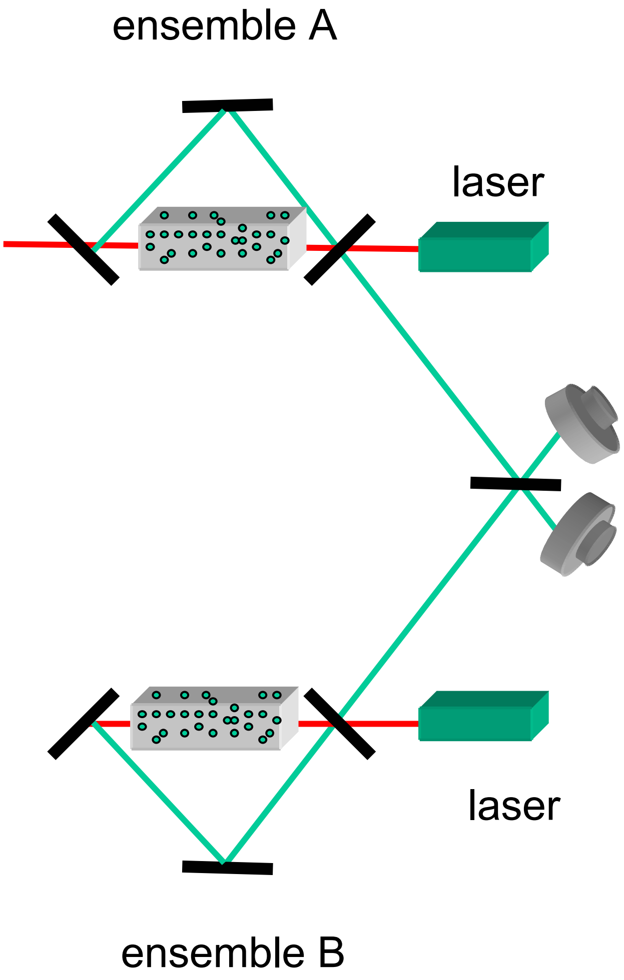
measurement gives

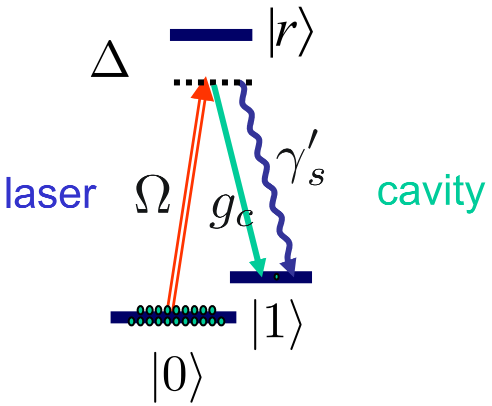
$$|\psi_{AB}^{\pm}\rangle = (a^\dagger \pm b^\dagger)|\text{vac}\rangle$$

$$\equiv |1_a, 0_b\rangle \pm |0_a, 1_b\rangle$$

We have generated entanglement between collective atomic states

Alternative model: atomic ensemble in a cavity



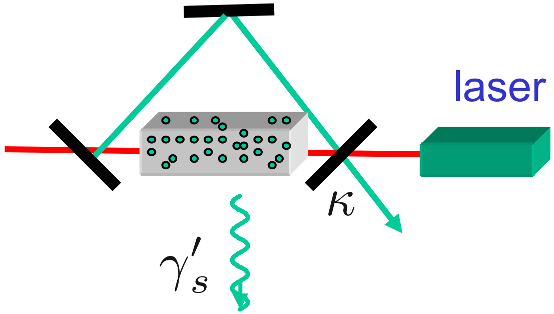


- Master equation: $\dot{\rho} = \ldots$

- Interaction with the laser
- Interaction with cavity mode
- Spontaneous emission.
- Cavity damping.

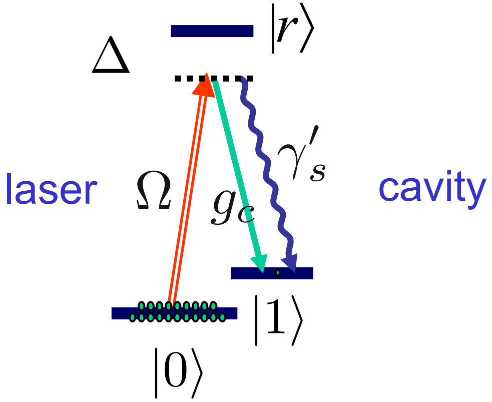
... an atomic ensemble improves the signal to noise ☺

Master equation: atoms in cavity



- master equation

$$\dot{\rho} = -i\left[H, \rho\right] + \Lambda\rho$$



- Hamiltonian

$$H = \hbar\frac{\sqrt{N_a}\Omega g_c}{\Delta} a^\dagger c^\dagger + \text{h.c.}$$

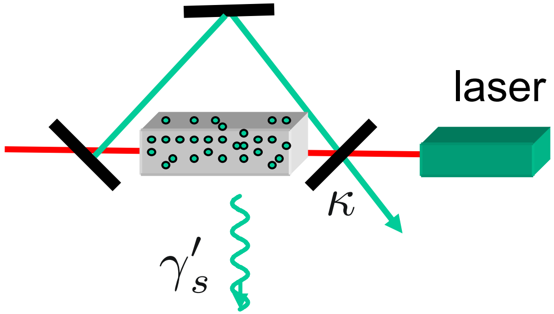
atoms

cavity

$$a \equiv \frac{1}{\sqrt{N_a}}\sum_{i=1}^{N_a} a_i \quad \left(a_i = |0\rangle_i \langle 1|\right)$$

$$|\phi\rangle \sim \sum_n \tanh r_c^n \frac{\left(c^\dagger\right)^n}{\sqrt{n!}} \frac{\left(a^\dagger\right)^n}{\sqrt{n!}} |0_a\rangle |0_p\rangle$$

two-mode squeezed atom + cavity state



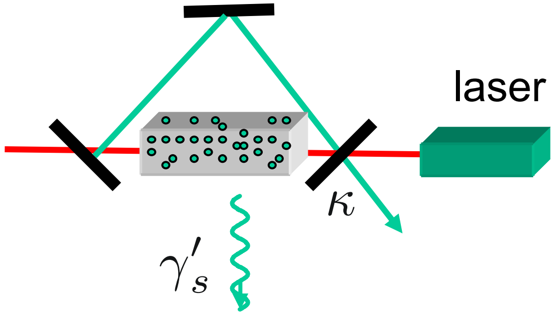
- cavity damping and spontaneous emission

$$\Lambda\rho = \frac{1}{2}\kappa(2c\rho c^\dagger - c^\dagger c\rho + \rho c^\dagger c) + \frac{1}{2}\gamma_s'\sum_i(2a_i^\dagger\rho a_i - a_i a_i^\dagger\rho - \rho a_i a_i^\dagger)$$

cavity damping

spontaneous emission

Master equation: bad cavity limit



- bad cavity limit $\kappa \gg \frac{\sqrt{N_a}\,|\Omega g_c|}{\Delta}$

- adiabatic elimination

$$\dot{\rho}_a = \frac{1}{2}\kappa'(2a^\dagger \rho_a a - a a^\dagger \rho_a - \rho_a a a^\dagger)$$

$$\kappa' = \frac{4N_a\,|\Omega g_c|^2}{\Delta^2 \kappa}$$

"good" Stokes emission

$$+\frac{1}{2}\gamma_s' \sum_i (2a_i^\dagger \rho_a a_i - a_i a_i^\dagger \rho_a - \rho_a a_i a_i^\dagger)$$

$$N_a \gamma_s'$$

bad spontaneous emission

- Q.: condition for good to bad ??

Master equation: collective atomic operators

- collective atomic operators

$$a_\mu \equiv \frac{1}{\sqrt{N_a}} \sum_{j=0}^{N_a} a_j e^{ij\mu\,/\,N_a} \qquad (\mu = 0, 1, \cdots, N_a - 1)$$

- master equation

$$\dot{\rho}_a \quad = \quad \frac{1}{2}\left(\kappa' + \gamma'_s\right)\left(2a^\dagger \rho_a a - a a^\dagger \rho_a - \rho_a a a^\dagger\right)$$

Stokes emission

good

bad

$$+\gamma'_s \sum_{\mu \neq 0} \xcancel{\left(2a^\dagger_\mu \rho_a a_\mu - a_\mu a^\dagger_\mu \rho_a - \rho_a a_\mu a^\dagger_\mu\right)}$$

other modes

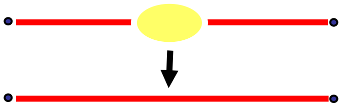
trace out!

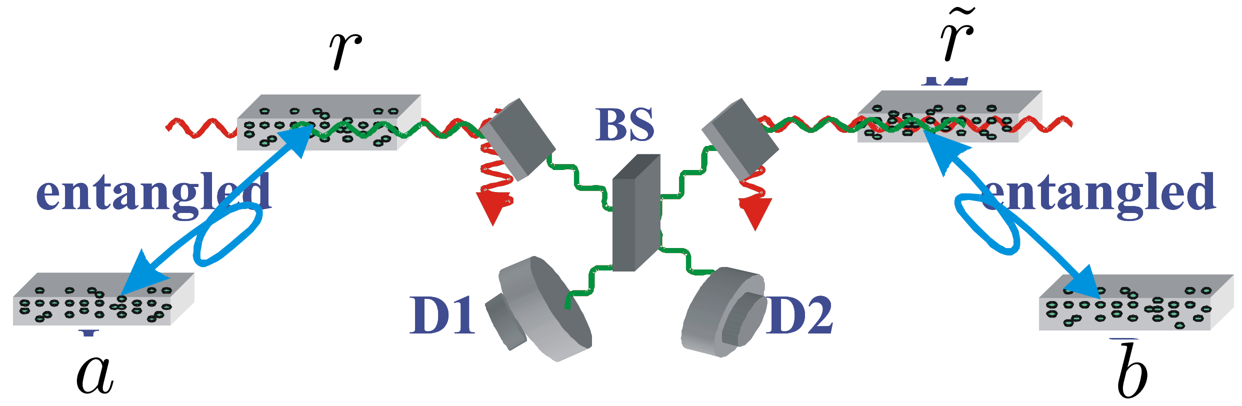
- signal to noise

$$R_{\mathrm{sn}} = \frac{\kappa'}{\gamma'_s} \sim \frac{4N_a \left|g_c\right|^2}{\kappa \gamma_s}$$

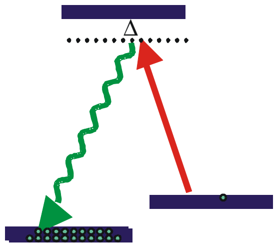
☺ ensembles help!

Connection





- steps
  - apply a red laser pulse to transfer atomic excitation to optical excitation



  - succeeds if D1 *or* D2 registers *one* photon: distance doubled!

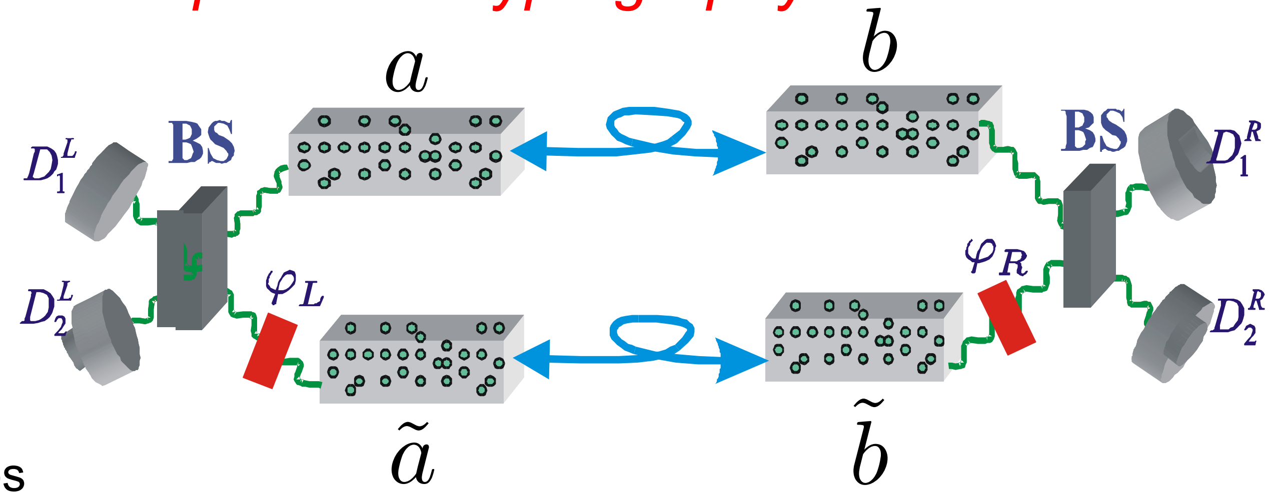
$$(a^\dagger + r^\dagger)(b^\dagger + \tilde{r}^\dagger)|\mathrm{vac}\rangle \longrightarrow (a^\dagger + b^\dagger)|\mathrm{vac}\rangle$$

(ideal)

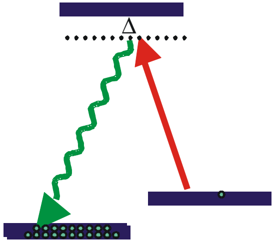
click=apply the operator:$(r+\tilde{r})$

  - fails otherwise: repeat everything starting from entanglement generation

Application: quantum cryptography



- steps
  - we generate two pairs
  - transfer atomic excitation to optical excitation, detect after phase shifter and beamsplitter



  - succeeds if D1 or D2 registers one photon on the left side and one photon on the right side.

$$(a^\dagger + b^\dagger)(\tilde{a}^\dagger + \tilde{b}^\dagger)|\mathrm{vac}\rangle \xrightarrow{\text{post selection}} (a^\dagger \tilde{b}^\dagger + \tilde{a}^\dagger b^\dagger)|\mathrm{vac}\rangle$$

equivalent to a photon polarization entangled state $\sim \quad |\updownarrow\longleftrightarrow\rangle + |\longleftrightarrow\updownarrow\rangle$

  - role of phase shifter: single-bit rotation

apply Ekert protocoll!

Imperfections:

- Spontaneous emission into other modes:

No effect, since they are not measured.

- Detector efficiency, photon absorption in the fiber, etc:

More repetitions.

- Dark counts:

More repetitions

- Technial note: anaylsis based on *effective maximally entangled state*

$$\rho = \frac{1}{c_0 + 1} \left(c_0 \left|\mathrm{vac}\right\rangle_{LR} \left\langle \mathrm{vac}\right| + \left|\Psi\right\rangle^{+}_{LR} \left\langle \Psi \right| \right)$$

✓ entanglement part decreases only linearly with L (instead of exponential)

✓ vacuum part drops out in quantum cryptography protocol

Scaling

- Fix the final fidelity: F

- Number of repetitions: $\sim L^{\log_2 L}$

- Example:

  Detector efficiency: 50%

  Length $L=100\ L_0$

  Time $T=10^6\ T_0$

  (to be compared with $T=10^{43}T_0$ for direct communication)

Conclusions

- Quantum repeaters allow to extend quantum communication to long distances.
- They can be implemented with trapped single atoms or atomic ensembles.
- The method proposed here is efficient and not too demanding:
    1. No trapping/cooling is required.
    2. No (high-Q) cavity is required.
    3. Atomic collective effects make it more efficient.
    4. No high efficiency detectors are required.

Mesoscopic atomic ensembles

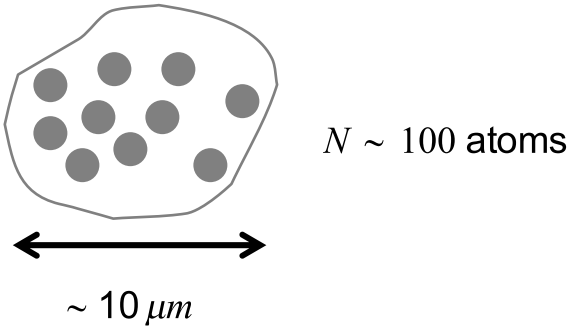
- idea
    - dipole blockade mechanism

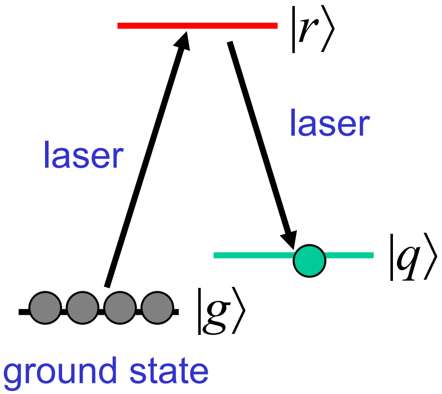
M. Lukin et al., PRL 2001

Configuration

- *mesoscopic* atomic ensembles (instead of microscopic quantum objects)

- coherent manipulation of *collective excitations* of atomic ensembles





-underlying physics:

dipole blockade

Manipulating collective excitations

- ground state

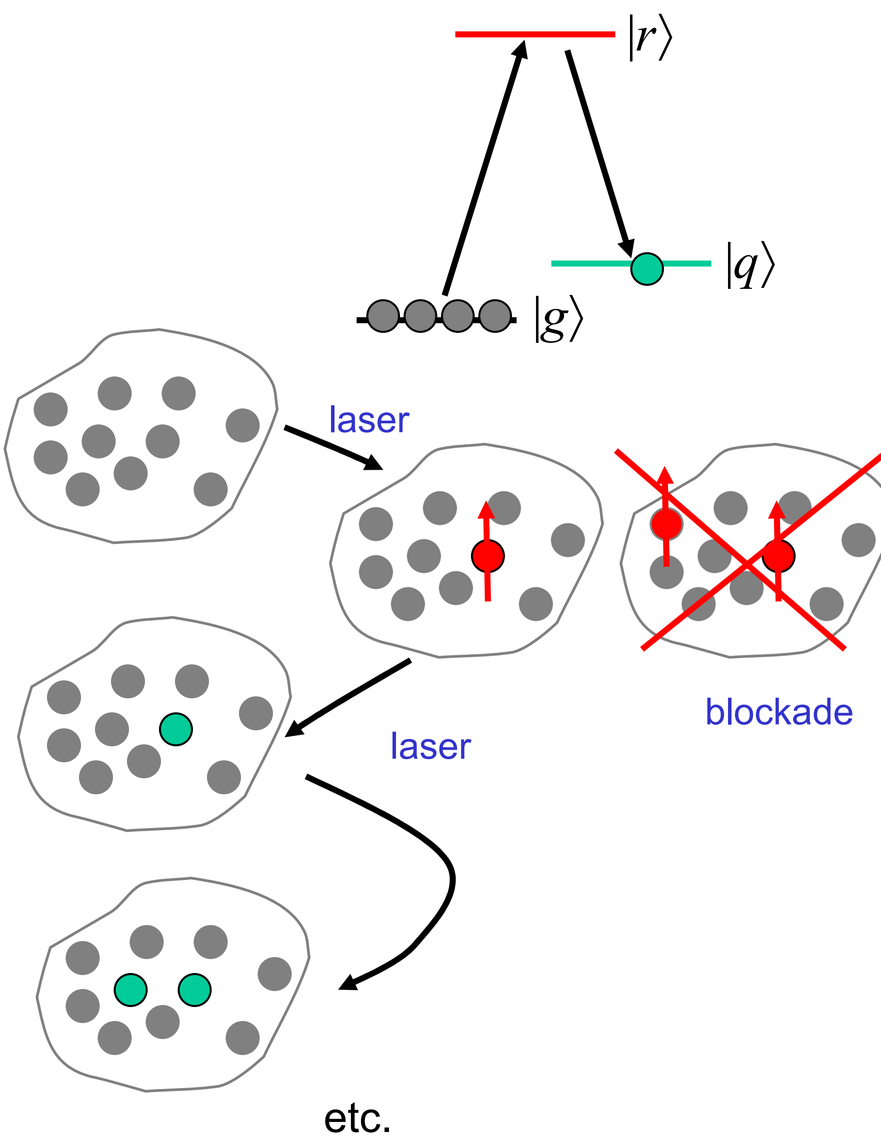
$$|g^N\rangle = |g_1\rangle|g_2\rangle\ldots|g_N\rangle$$

- one excitation (Fock state)

$$|g^{N-1}q\rangle \sim \sum_i |g_1\rangle\ldots|q_i\rangle\ldots|g_N\rangle$$

- two excitations

$$|g^{N-2}q\rangle \sim \sum_{i,j} |g_1\rangle\ldots|q_i\rangle\ldots|q_j\rangle\ldots|g_N\rangle$$

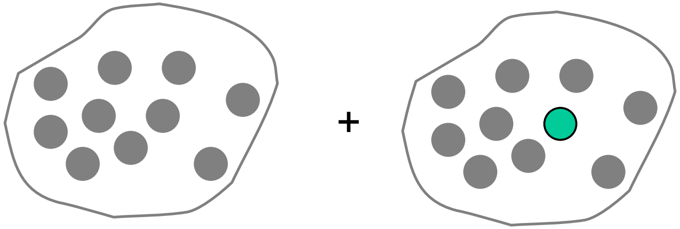


We can store and manipulate qubits.

cont.

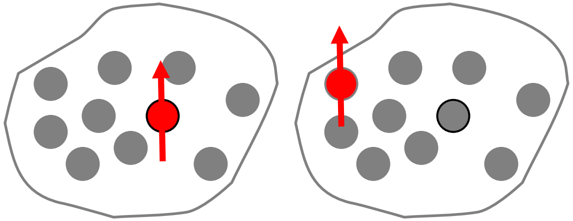
- qubits

$$|\psi\rangle = \alpha|g^N\rangle + \beta|g^{N-1}q\rangle$$



superposition

- entanglement of ensembles



Teleporation with coherent light and ensembles

L, M Duan et al., PRL 2000; exp: E. Polzik et al. Nature 2001

Continuous Variable Teleportation

- Instead of qubits we consider now continuous variable quantum states

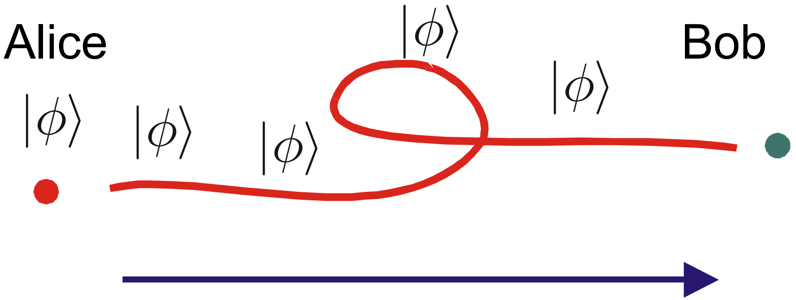
$$|\phi\rangle = \int dx\, |x\rangle\, \phi(x)$$

$\hat{x}$ … position

$\hat{p}$ … momentum

$$[\hat{x},\hat{p}] = i$$

- transmission of a cv state



- continuous variable teleportation

Vaidman
Braunstein
Kimble (exp)

$$|\mathrm{EPR}\rangle_{AB} \sim \int dx\, |x\rangle_A |x\rangle_B$$

$$\sim \int dp\, |p\rangle_A |-p\rangle_B$$

$$(\hat{x}_A - \hat{x}_B)|\mathrm{EPR}\rangle = x_1 |\mathrm{EPR}\rangle$$

$$(\hat{p}_A + \hat{p}_B)|\mathrm{EPR}\rangle = p_1 |\mathrm{EPR}\rangle$$

Teleportation with Squeezed Light

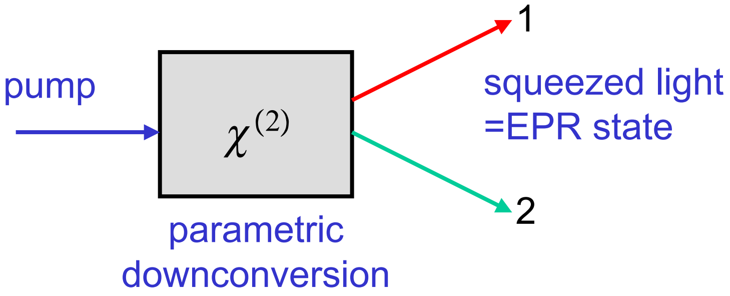
S. Braunstein, H.J. Kimble et al., PRL '98; Science '99

- Two-mode squeezed light:

electric field $E^{(+)} \sim a\, e^{ikx-i\omega t}$

$= \hat{x} + i\hat{p}$

quadrature components

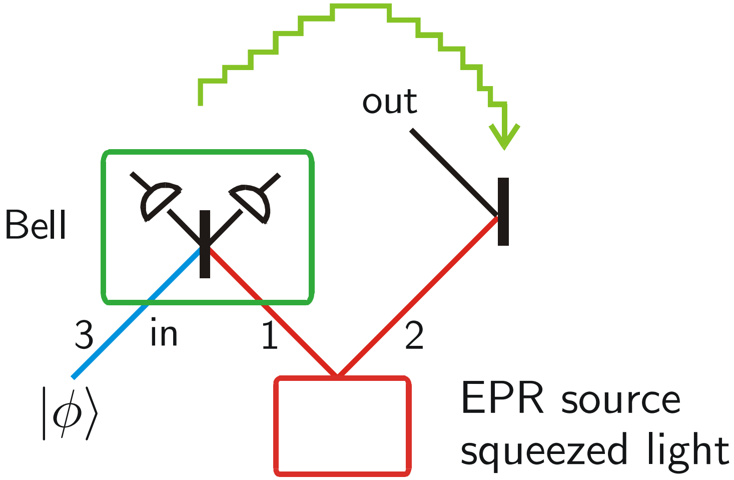


pump

squeezed light
=EPR state

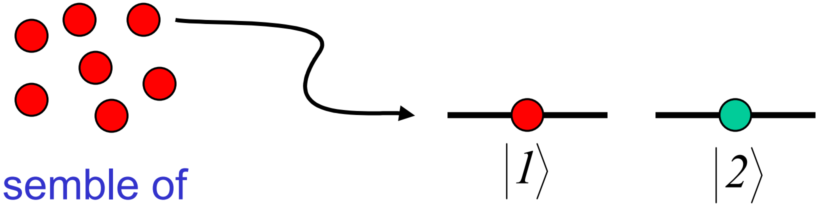
parametric
downconversion

- Scheme



Atomic ensembles as quantum memory for cont var states

- We consider an ensemble of N atoms



ensemble of atoms

two-level atom =spin -1/2

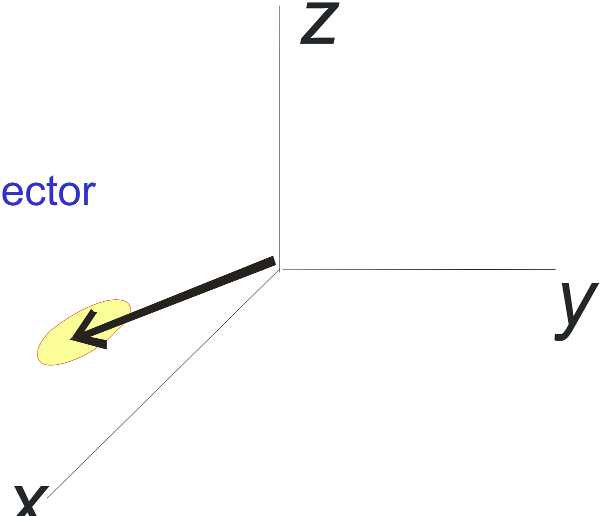
- a collection of two-level atoms can be described in terms of a collective “angular momentum“

$$\vec{S}^a = \sum_{\mu=1}^{N} \frac{1}{2}\vec{\sigma}^{(\mu)}$$

collective angular momentum

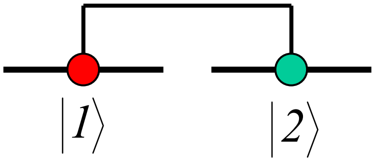
two-level atom = spin -1/2

Bloch vector



atoms cont.

- superposition of the two ground states: coherent spin state



$$\left[\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)\right]^{\otimes N}$$

Bloch vector

$$\langle \vec{S}^a \rangle = (\langle S_x^a \rangle, \langle S_y^a \rangle, \langle S_z^a \rangle) = (\frac{N_a}{2}, 0, 0)$$

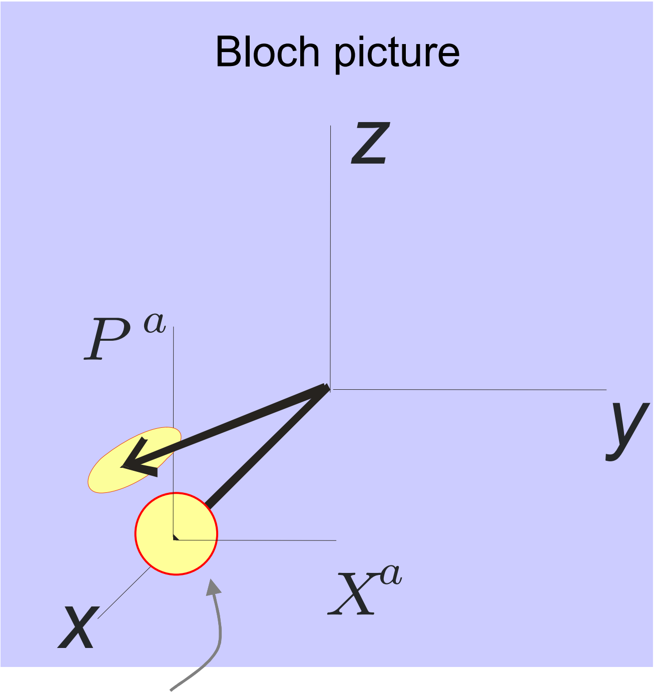
- quantum fluctuations

$$[S_y^a, S_z^a] = iS_x^a \qquad \Delta S_y^a \Delta S_z^a \geq \frac{1}{2}|\langle S_x^a \rangle|$$

we treat S_x^a classically and rescale

$$[X^a, P^a] = i \qquad \Delta X^a \Delta P^a \geq \frac{1}{2}$$

canonical communation relations

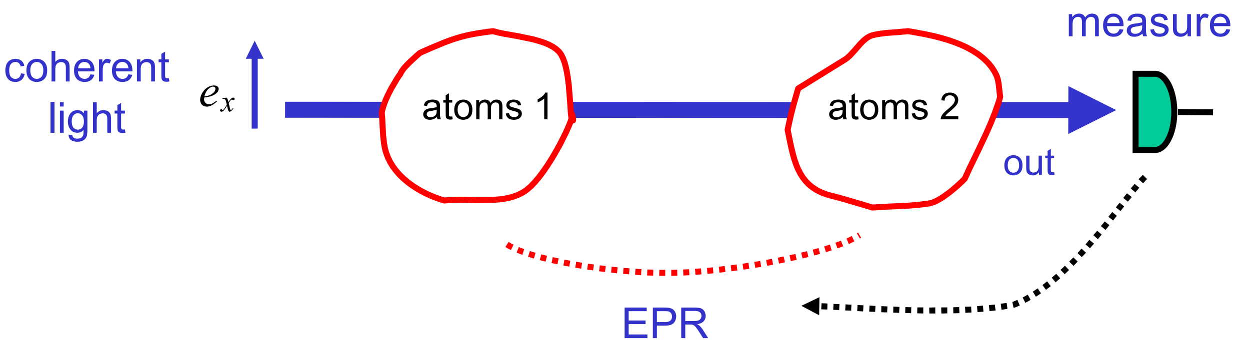


✓ coherent spin state =vacuum state

✓ there are *many* cv quantum states around it:

$$|\psi^a\rangle = \int dX^a |X^a\rangle \psi(X^a)$$

Teleportation with coherent light + atomic ensembles



measurement projects atomic ensembles into continuous variable EPR state

$$|\mathrm{EPR}\rangle \sim \int dP \, |P\rangle_A |-P\rangle_B = \int dX \, |X\rangle_A |X\rangle_B$$

- ✓ theory: Innsbruck
- ✓ experiment: E. Polzik et al. (Aarhus), Nature 2001